CHARACTERISTICS OF ENTRAINMENT OF A CONDUCTING LOOP BY A MOVING MAGNETIC FIELD

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The problems of accelerating macroparticles with sizes of several millimeters up to velocities of 10^6-10^8 cm/sec are currently of great interest for the physics of high-temperature plasma [1]. Thus, one of the most promising methods for replenishing a thermonuclear reactor with fuel is injecting tablets of thermonuclear fuel into the reaction zone with the required rate. One possible method for obtaining a thermonuclear reactor with inertial confinement is pulsed heating accompanying high-speed collision of macroparticles with a target, when the particle or target are prepared from thermonuclear material.

Estimates in [1] show that a possible method for accelerating a conducting or superconducting composite granule of deuterium is inductive acceleration in a magnetic field with the interaction of this field with the dipole magnetic moment induced in the granule. An elementary estimate of the limiting velocity for a spherical diamagnetic particle is given in [1, 2].

In this paper, we examine the problems of capturing and entraining a stationary conducting body, shaped like an annular loop and moving in a magnetic field, within the framework of the theory of electrical circuits for moving loops in application to the problem of accelerating macroparticles. Investigations of simple working models reveal the important characteristics of the process by which a conductor is captured by the magnetic-field wave. The quantitative estimates obtained can be used to analyze the acceleration of a perfectly conducting body or a body with finite electrical conductivity, whose size in the direction of diffusion of the magnetic field is less than the penetration depth of the field (thin conductor).

Formulation of the Problem. Regarding the inductor system, which creates the moving magnetic-field wave, as ideally distributed, we shall assume that a current zone moves along its coils. When the supply circuits of the coils are independent, this current zone in some approximation can be viewed as an electrically conducting loop (Fig. 1) moving with constant velocity v_1 = const and initial current i_{10}, which encounters a loop 2 at rest, i.e., the body being accelerated. As the loops approach, due the interaction of the magnetic field of the moving loop and the magnetic dipole moment induced in loop 2, the latter is dragged along. We shall describe the magnetic coupling between the loops by the coupling coefficient $k(x) = M/\sqrt{L_1L_2}$, where x is the distance between the loops, M(x) is the mutual inductance, while L_1 and L_2 are the inductance of the first and second loops, respectively. For $k \to 1$, the loops approach in such a way that the magnetic leakage fluxes between them approach zero.

1. We shall first examine the case when the active resistances of both loops are negligibly small. Starting from the constancy of the flux linkages, penetrating the loops, we obtain $\Psi_1 = L_1i_1 + Mi_2 = L_1i_{10}$, $\Psi_2 = L_2i_2 + Mi_1 = 0$, from where $i_1 = i_{10}/[1 - k^2(x)]$. Then, the magnetic energy of the system of two loops equals $W_{12} = W_{10}/[1 - k^2(x)]$, where $W_{10} = L_1i_{10}^2/2$ is the initial magnetic energy of the first loop. Let us write out, using the expression for W_{12}, the equation of energy balance for the two loops in a system of coordinates fixed to the moving loop:

$$m_2v_1^2/2 + W_{10} = m_2v^2/2 + W_{10}/(1 - k^2(x)), \tag{1.1}$$

where $v = v_2 - v_1$ is the velocity of the second loop relative to that of the first; m_2 is the mass of the second loop.

It is evident that for small values of W_{10} loop 2 can pass through loop 1 (we use a system of coordinates fixed to the first loop), while for large values of W_{10} loop 2 can be reflected from loop 1. The second case (the case of total entrainment of the second loop) is

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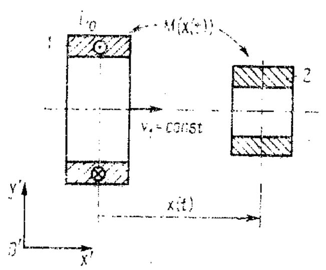

Fig. 1

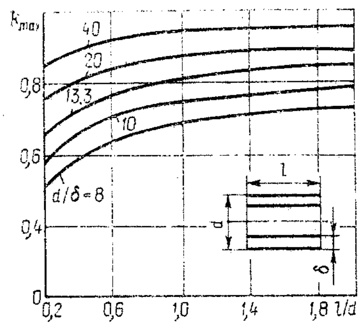

Fig. 2

characterized by the fact that at a well-defined time, preceding reflection and corresponding to a well-defined value of k(x), the loops will be stationary relative to one another (v = 0). In this case, we obtain from (1.1)

$$m_2 v_1^2/2 = W_{10} k^2(x)/(1 - k^2(x)). \tag{1.2}$$

The right side of expression (1.2) assumes its maximum value for $k(x) = k_{max}$, i.e., at the time of exact coincidence of the loops. The condition for total entrainment of the second loop can be written in the form

$$W_{10} k_{max}^2/(1 - k_{max}^2) \geqslant m_2 v_1^2/2. \tag{1.3}$$

It follows from equality (1.1) that in the case of an elastic "collision" of the loops, when condition (1.3) is satisfied, loop 2 by the end of the interaction will be moving away from loop 1 with velocity v_1. In addition, its final velocity in a stationary system of coordinates is $v_2^* = 2v_1$. Thus, if it is necessary to accelerate a conducting loop with mass m_2 up to velocity v_2^* in the case of negligibly small dissipative processes, the magnetic field of the inductor system must move with velocity $v_1 = 0.5\, v_2^*$ and have magnetic energy that satisfies, according to (1.3), the condition

$$W_{10} \geqslant \frac{1}{4} \frac{1 - k_{max}^2}{k_{max}^2} \frac{m_2 v_2^{*2}}{2}. \tag{1.4}$$

To estimate the magnitude of k_{max}, in many cases, it is possible to use the calculations of k_{max} for two concentric infinitely thin solenoids (Fig. 2). For example, for $k_{max} = 0.9$, $W_{10} \geqslant 0.06 m_2 v_2^2/2$.

2. Let us examine the entrainment of a conducting loop 2 with an active resistance R_2 for the preceding assumptions concerning the inductor system. Assume that loop 2 represents a loop moving with velocity v_1 = const and with initial current and active resistance $R_1 = 0$. In this case, the electromechanical transient process in a system of coordinates fixed in the first loop is described by the following system of nonlinear differential equations, written for dimensionless quantities:

$$\frac{d\tilde{i}_2}{d\tilde{t}} = \frac{1}{1 - k^2(\tilde{x})}\left[\tilde{v}\tilde{i}_2 \frac{dk^2(\tilde{x})}{d\tilde{x}} - \rho\tilde{i}_2 - \tilde{v}\frac{dk(\tilde{x})}{d\tilde{x}}\right], \tag{2.1}$$

$$\frac{d\tilde{v}}{d\tilde{t}} = \frac{1}{\mu}\tilde{i}_2[1 - k(\tilde{x})\tilde{i}_2]\frac{dk(\tilde{x})}{d\tilde{x}}, \quad \tilde{i}_1 = 1 - k(\tilde{x})\tilde{i}_2,$$

$$\frac{d\tilde{x}}{d\tilde{t}} = \tilde{v}, \quad \tilde{i}_2|_{\tilde{t}=0} = 0, \quad \tilde{x}|_{\tilde{t}=0} = 1, \quad \tilde{v}|_{\tilde{t}=0} = -1,$$

where $\tilde{x} = x/b$ (b is the initial, in general, arbitrarily chosen distance between the loops); $\tilde{i}_1 = i_1/i_{10}$; $\tilde{i}_2 = \sqrt{L_2 i_2^2/L_1 i_{10}^2}$; $\tilde{v} = v/v_1$; $\tilde{t} = v_1 t/b$; $\rho = R_2 b/L_2 v$ is the relative resistance of the second loop; and $\mu = m_2 v_1^2/L_1 i_{10}^2$ is the relative mass of the second loop. The following approximate dependence of the coupling coefficient on the distance between loops was used: $k(x) = k_{max} \exp(-3|x|)$.

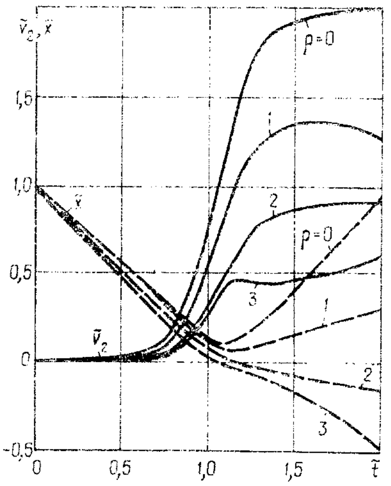

Fig. 3

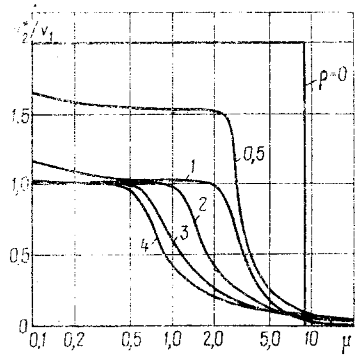

Fig. 4

The system (2.1) was integrated numerically using the Runge—Kutta method. The distance between the loops $\tilde{x}(t)$ and the velocity of the second loop $\tilde{v}_2 = 1 + \tilde{v}$, calculated in the stationary system of coordinates, as a function of time are presented in Fig. 3 ($\mu = 1$ and $k_{max} = 0.95$). For $\rho = 0$, the transfer of a mechanical momentum via the electromagnetic field from the moving first loop to the initially motionless second loop is elastic. When the conditions for entrainment (1.3) are satisfied, the first loop does not overtake the second loop, finally acquiring as a result of the interaction a velocity that is twice its initial value. As the relative resistance of the second loop ρ increases, momentum exchange becomes inelastic: the final velocity of the second loop decreases. Figure 4 ($k_{max} = 0.95$) shows the effect of the relative mass μ and resistance ρ of the second loop on the magnitude of its final velocity v_2^*. The final velocity is understood to mean the velocity of the second loop in the stationary system of coordinates upon completion of its interaction with the first loop, when $k \to 0$. For $\rho = 0$, the loop being accelerated can either be accelerated to a velocity $v_2^* = 2v_1$ ($\mu < \mu_*$), or it can arrive at a state of rest ($\mu > \mu_*$). Starting from the condition (1.2), we find the critical values of the mass $\mu_* + k_{max}^2/(1 - k_{max}^2)$. In the presence of Joule dissipation, the magnitude of the critical mass of the second loop decreases. In this case, for $\mu > \mu_*$, the first loop overtakes the second loop, whose velocity no longer decreases to zero at the end, as in the case $\rho = 0$, and the second loop continues to move with some velocity $v_2 < v_1$, partially entrained by the magnetic field of the first loop (Fig. 3, $\rho = 3$). The partial entrainment of the second loop can be explained by the fact that after the time that the contours coincide, the current in the second loop changes sign, which is not observed for $\rho = 0$. This is evident from Fig. 5, where the time dependences of the currents in the loops during their interaction are illustrated ($\mu = 1$, $k_{max} = 0.95$).

In conclusion, we note the following.

1. To accelerate a perfectly conducting body to a given velocity with the help of a traveling magnetic wave, the velocity of the wave must be equal to one-half the velocity of the body and the condition (1.4) for the body to be captured by the wave must be satisfied.

2. The condition for capture of a perfectly conducting body by a traveling magnetic field is the requirement that the energy in the magnetic field carried by the wave, taking into account the coupling coefficient with the body being accelerated, be sufficient for imparting to the body a kinetic energy corresponding to the motion of a body with the velocity of the wave.

3. The transfer of mechanical momentum from the traveling magnetic wave to the body being accelerated is elastic if the body is perfectly conducting and its relative mass is less than the critical mass. In this case, the final velocity of the body exceeds by a factor of 2 the velocity of the magnetic wave.

4. The acceleration of a body with finite electrical conductivity by a traveling magnetic wave has the nature of an inelastic interaction, whose measure is the relative active resistance of the body. In this case, the concept of the critical mass of the body, above which the entrainment of the body by the magnetic wave terminates, remains meaningful. As

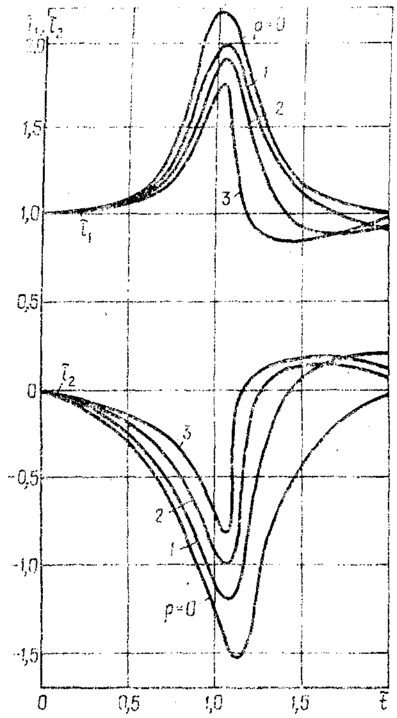

Fig. 5

the active resistance increases, the critical mass and the final velocity of the body accelerated decrease.

5. The magnitude of the magnetic field energy transported by the wave, required to accelerate a body or macroparticle, depends strongly on the maximum attainable value of the coupling coefficient between the coils of the accelerating field and the particle ($W_M \sim (k_{max}^{-2} - 1)mv^2/2$).

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